Stellar Spectrum Synthesis

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Contents

1 Basics

1.1 Equation of radiative energy transport

Radiation transports energy through a stellar atmosphere. The simple approximation of a plane parallel layering holds for relatively thin atmospheres

$$\frac{dI_\nu}{ds} = j_\nu - \kappa_\nu I_\nu \tag{1}$$

Here j_ν denotes the emission and κ_ν the absorption per Volume element. We define the source function as

$$S_\nu = \frac{j_\nu}{\kappa_\nu} \tag{2}$$

Together with the optical depth

$$d\tau_\nu = -\kappa_\nu dz \tag{3}$$

$$\rightarrow d\tau_\nu = -\kappa_\nu cos\theta ds \tag{4}$$

$$ds = -\frac{1}{\kappa_\nu cos\theta} d\tau_\nu \tag{5}$$

we can write the simplified equation of radiative energy transport

$$-\frac{dI_\nu}{d\tau_\nu}\kappa_\nu cos\theta = j_\nu - \kappa_\nu I_\nu \tag{6}$$

$$-cos\theta\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu \tag{7}$$

and its formal solution

$$I_\nu = -e^{-\frac{\tau_\nu}{cos\theta}} \int_{const}^{\tau_\nu} S_\nu e^{-\frac{t}{cos\theta}} \frac{dt}{cos\theta} \tag{8}$$

The intensity of the outward directed radiation (all contributions of $\tau = C$ bis $\tau = \infty$) at a certain depth $\tau\nu = C$ is therefor

$$I_\nu(cos\theta, C) = -e^{-\frac{C}{cos\theta}} \int_C^\infty S_\nu e^{-\frac{\tau_\nu}{cos\theta}} \frac{d\tau_\nu}{cos\theta} \tag{9}$$

This means, if we know the source function S_ν as function of τ_ν, we kann easily calculate the intensity of the emitted energy spectrum.

1.2 Local Thermal Equilibrium (LTE)

Let us assume that we have local thermal equilibrium inside our stellar atmosphere. In this case we can express the level populations (i.e. probability a level is occupied), n_i, n_j, of two energy levels of a species as following

$$\frac{n_i}{n_j} = \frac{g_i}{g_j} e^{-\frac{\Delta E_{ij}}{kT}} \tag{10}$$

Here g_i and g_j are the statistical weights of these two energy levels. In the case that one of these two levels is the continuum (0–level), we call the energy difference ΔE_{iC} the excitation energy (χ_i) of the state i. It is clear that the number of particles is preserved

$$n = \sum_i n_i = \frac{n_C}{g_C} \sum g_i e^{-\frac{\chi_i}{kT}} = \frac{n_C}{g_C} U_n \tag{11}$$

The sum U_n generally is called the 'partition function' or the 'partition sum' of n.

1.3 Continuum source function

We can separate the contributions of spectral lines ($\kappa_{\nu L}$, $j_{\nu L}$) and continuum ($\kappa_{\nu C}$, $j_{\nu C}$) to the energy balance and also to the source function.

$$j_\nu = j_{\nu C} + j_{\nu L} \tag{12}$$

$$\kappa_\nu = \kappa_{\nu C} + \kappa_{\nu L} \tag{13}$$

$$S_\nu = \frac{\kappa_{\nu L}}{\kappa_\nu} S_{\nu L} + \frac{\kappa_{\nu C}}{\kappa_\nu} B_\nu \tag{14}$$

The reason for doing so is that now we can substitute the Plank function B_ν for the continuum part of the source function. Thus, we are left only with S_L to calculate the emitted radiation intensity. In many modern synthesis codes this is not done, but time is saved by solving $S_{\nu C}$ on a much wider grid.

1.4 Atomic lines

Now we concentrate on the contribution of the atomic lines to the source function. For convenience we will omit from now on the suffix '$_L$' knowing that we are dealing with spectral lines. First some definitions

Φ_ν ... line absorbtion profile; norm: $\int \Phi_\nu d\nu = 1$
Ψ_ν ... linie emission profile; norm: $\int \Psi_\nu d\nu = 1$
A_{ji} ... probability for spontaneous emission (j$\mapsto$i)
B_{ij} ... absorbtion coefficient (j$\mapsto$i)
B_{ji} ... probability for induced absorbtion (i$\mapsto$j)

Using above quantities we can express S_L as follows

$$S_{\nu L} \text{ now } S_\nu = \frac{j_\nu}{\kappa_\nu} = \frac{h\nu A_{ji} n_j \Psi_\nu}{h\nu(B_{ij}n_i - B_{ji}n_j)\Phi_\nu} = \frac{A_{ji}n_j}{B_{ij}n_i - B_{ji}n_j}\frac{\Psi_\nu}{\Phi_\nu} \tag{15}$$

Furthermore holds

$$g_i B_{ij} = g_j B_{ji} \tag{16}$$

$$A_{ji} = \frac{2h\nu^3}{c^2} B_{ji} \tag{17}$$

Putting this into above equation we get a general form for the contribution of atomic lines to the source function

$$S_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\frac{n_i g_j}{n_j g_i} - 1}\frac{\Psi_\nu}{\Phi_\nu} \tag{18}$$

1.5 Absorbion profile

As shown in the previous subsection the line absorbtion κ_ν depends on the absorbtion coefficient B_{ij} and the absorbtion profile Φ_ν

$$\kappa_\nu = h\nu B_{ij}(n_i - n_j\frac{g_i}{g_j})\Phi_\nu \tag{19}$$

The general shape of the absorbtion profile then again does not vary much from line to line. In the following we will discuss the basic broadening mechanisms in stellar atmospheres.

Doppler broadening ($\Delta\lambda_D$)
Thermal (particle movement) line broadening because of the Doppler effect

$$\Delta\lambda_D = \frac{\lambda_L}{c}\sqrt{\frac{2kT}{m_n} + v_{micro}^2} \tag{20}$$

where
λ_L ... central wavelength of the spectral line
m_n ... mass of the atom, ion

Radiadive damping (γ_{rad})

$$\gamma_{rad} = \sum_{l<i} \frac{1}{\text{lifetime } n_i} + \sum_{l<j} \frac{1}{\text{lifetime } n_j} \tag{21}$$

Collisional damping: Stark effect (γ_{st})
Collisions with other ions and electrons

$$\gamma_{st} \sim \Delta\lambda^{-2} \text{ ... Stark effect (H \& He)} \tag{22}$$
$$\gamma_{st} \sim \Delta\lambda^{-4} \text{ ... quadratic Stark effect} \tag{23}$$

Collisional damping: Van der Waals (γ_{vw})
Collisions with neutral atoms

$$\gamma_{vw} \sim \Delta\lambda^{-6} \tag{24}$$

Radiative and collisional damping are usually combined to one damping constant γ

$$\gamma = \gamma_{rad} + \gamma_{st} + \gamma_{vw} \tag{25}$$

1.5.1 Voigt Profil

Let us take a closer look on the absorption process in a gas on microscopic level. We have to average over an essemble of atoms the case of a photon absorbed by a single atom. Because the atoms move around in the gas the probability for a photon to be absorbed depends not only on the energetic difference of the atomic levels involved (ΔE_{ij}) but also on the velocities and directions of motion of the atoms. Be that the velocity distribution is Maxwell'sh and our absorbtion profile Φ satisfies the normalization

$$\int_{-\infty}^{\infty} \Phi(\nu) d\nu = 1 \tag{26}$$

we get in direction v_z (see also Cannon, [1985])

$$\Phi(\Delta\nu) = \frac{\gamma}{\pi^{3/2}(\Delta\nu_D)^2} \int_{-\infty}^{\infty} \frac{e^{\frac{v_z^2 m_n}{2kT}}}{(\frac{\Delta\nu}{\Delta\nu_D} - \frac{v_z\sqrt{m_n}}{\sqrt{2kT}})^2 + (\frac{\gamma}{\Delta\nu_D})^2} d(\frac{v_z\sqrt{m_n}}{\sqrt{2kT}}) \tag{27}$$

or using the Voigt function

$$\Phi(\Delta\nu) = \frac{1}{\sqrt{\pi}\Delta\nu_D} H(a, \frac{\Delta\nu}{\Delta\nu_D}) \tag{28}$$

Here we have $a = \frac{\gamma}{\Delta\nu_D}$ and the Voigt function

$$H(a,\eta) = \frac{a}{\pi}\int_{-\infty}^{\infty} \frac{e^{-y^2}}{(y-\eta)^2 + a^2)} dy \tag{29}$$

If we are not dealing with the very strong lines then $a << 1$. In the line center, which is for small $\Delta\nu$ we can simplify Φ to

$$\Phi(\Delta\nu) = \frac{1}{\sqrt{\pi}\Delta\nu_D} e^{-(\frac{\Delta\nu}{\Delta\nu_D})^2} + \text{terms of higher order} \tag{30}$$

Whereas for the line wings (large $\Delta\nu$) the equation for Φ simplifies to

$$\Phi(\Delta\nu) = \frac{a\Delta\nu_D}{\pi(\Delta\nu)^2} + \ \ldots \ = \frac{\gamma}{\pi(\Delta\nu)^2} + \ \ldots \tag{31}$$

1.6 Oszillator strengths

The transitions between the energetic niveaus are to be treated by the theory of quantum mechanics. Thus the classical energy levels E_i and E_j are fuzzy

$$\Delta E \cdot \Delta t = \hbar \ \ldots \text{Heisenberg} \tag{32}$$

The form of classical equations stays valid, but they have to be corrected when we change from the classic transition probabilities to their quantum-mechanical equivalents. To account for this one introduces oscillator strenghts

$$f_{ij} = \frac{m_n h\nu}{\pi e^2} B_{ij} \tag{33}$$

1.7 Theoretical curve of growth

Now we combine the equations we derived for κ, B_{ij}, n_i and Φ. Furthermore we leave the frequency domain and transform the result into the wavelength domain, $\Delta\nu \mapsto \Delta\lambda$

$$\kappa(\Delta\lambda) = \frac{h}{\lambda} B_{ij} n_i (1 - \frac{n_j g_i}{n_i g_j}) \Phi(\Delta\lambda) \tag{34}$$

We can approximate the line core by

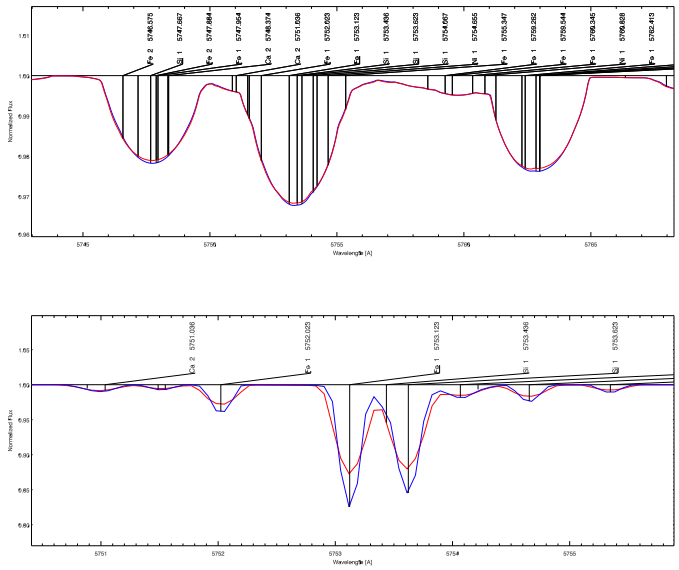
$$\kappa(\Delta\lambda) = Const. \cdot f_{ij} \cdot n_i \cdot \frac{\lambda_L}{\Delta\lambda_D} \cdot e^{-(\frac{\Delta\lambda_L}{\Delta\lambda_D})^2} \tag{35}$$

and the wings

$$\kappa(\Delta\lambda) = Const. \cdot f_{ij} \cdot n_i \cdot \gamma \cdot \frac{\lambda_L^4}{\Delta\lambda^2} \tag{36}$$

2 Geometric effects

Stars are not flat, but if they are not rotating to fast we can approximate them as spheres. For example the synthesis code synth3 (Kochukhov, [2006]) solves the transfer equation for seven angles (i.e. seven rings). These contributions to the total intensity spectrum are folded with instrumental, rotation an turbulence profiles and integrated over the stellar disc.

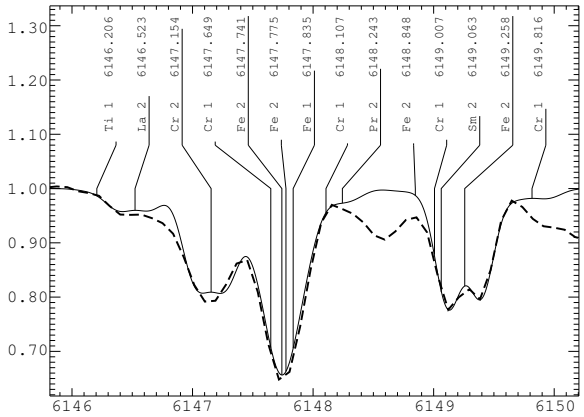


2 line profiles showing the difference between spherical approximation (red) and simple plane parallel aproximation (blue) in spectrum synthesis. Upper figure $v\sin i$ = 100 km/s. Lower figure $v\sin i$ = 10 km/s.

Working with straight forward plane parallel approximation may, as illustrated in the above figure, introduce quite large systematic errors when analysing slower rotators. Fast rotators however set a different task since effective temperature and surface gravity are not constant over the stellar surface.

3 Magnetic fields

Magnetic fields add more broadening mechanisms, fine structure and hyperfine structure, known as the Zeeman effect. Naturally the transfer equation gets more complex and computation times considerably longer. The interpretation of observed line profiles for the case of large, global magnetic fields are quite tricky and requires usually more than just intensity profiles, since not only the strength but also the geometry and orientation of the field has to be considered.



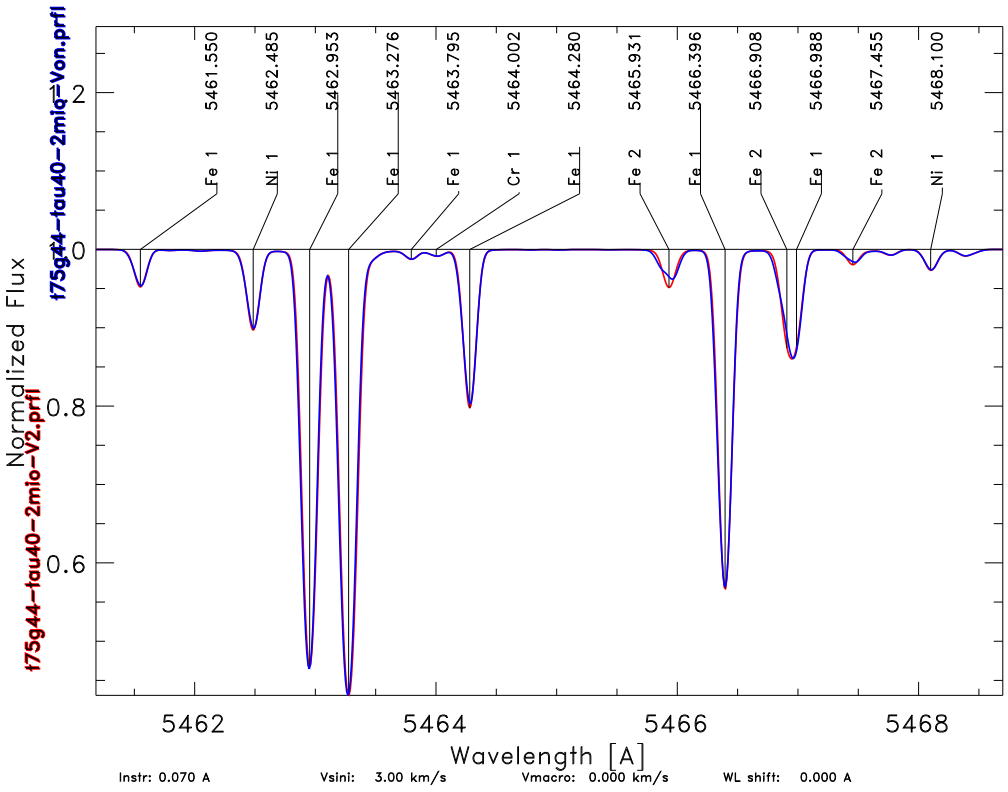
Two FeII lines in the spectrum of HD 18610 exhibiting a global magnetic surface field of the order of 7 kG. The FeII line at 6149.258 Å shows Zeeman splitting, the other, FeII 6147.649, is rather insensitive to the magnetic field.

4 Velocity fields

Another property of stellar atmospheres that distorts the shape of spectral lines is turbulent motion. Convective energy transport is a highly non-local process which cannot be descriped properly by only algebraic equations. In order to do so one needs to solve the Navier Stokes Equations, a non closed set of differential equations, or numerically simulate part of the stellar atmosphere and envelope; a task which is still to time expensive in practical spectroscopy. Thus several fudges, parametrizations and convection models are in use: The mixing length theory, where all the kinetic energy is put into one convection cell of a characteristic size. The non-locality is ignored. Another equally fast convection treatment was introduced by Canuto ([1992], where this characteristic size varies with depth. Basicly all stellar evolution

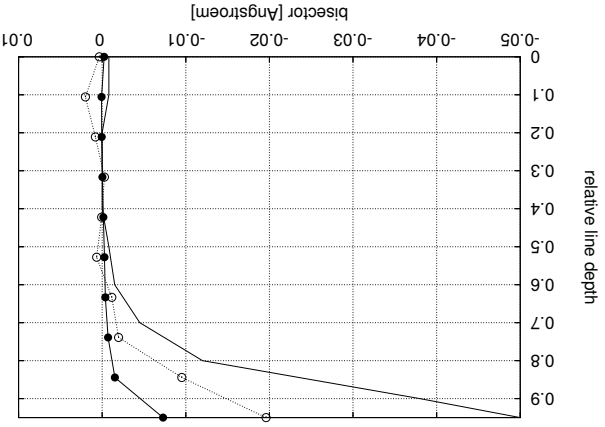
codes up to now use such a convection modelling. Turbulent velocity and scale are specified as fixed parameters. Also in spectrum synthesis mostly this approach is followed. Spectroscopists usually work with microturbulence (small scale velocity field) and macroturbulence (large scale).

But contrary to this arbitrary parametrized symetrical broadening, high resolution observations indicate line asymetries (bent bisectors) in various types of stars.



Above figure shows a comparison between a synthesis broadened with the final value for $v_{\rm mic}$ is 2 km/s (red) and a parameter free synthesis from a velocity dynamics model (blue). Both are convolved with instrumental and a small rotation profile. They agree very well, but one notices that line asymetries cannot not be (re)produced with the parametrization.

Bisectors for a star. Full line are measurements of one line, the open circles show the average bisector of about ten selected atomic lines, the filled circles are computations from synthetic line profiles of the same lines.



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