Models of Stellar Atmospheres

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Prerequisites...

- In 21th century we still get most of the information via analysis of electromagnetic radiation
- 80% of electromagnetic radiation is coming from stars

Analyzing electromagnetic radiation we should understand that:

- We directly analyze only the region where the radiation becomes the one we register via our telescopes. This region is transparent for the radiation to leave the star and is called stellar atmosphere.
- Everything that is taking place deeper the stellar atmosphere we can analyze only indirectly

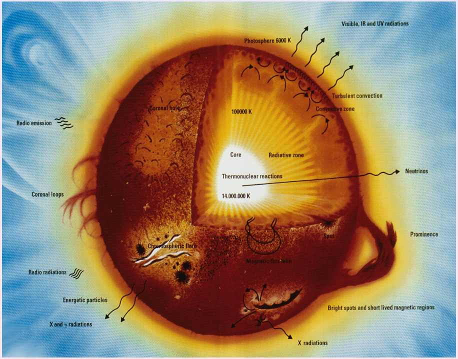
Definition: Stellar Atmosphere

General

Stellar atmosphere: it is a part of the star which does not have its own energy sources. Only redistribution of radiative energy takes place.

Practical

Stellar atmosphere: it is a part of the star where the line spectra is formed (outcoming radiation which we observe)



Definition: Model of Stellar Atmosphere

Since the radiation we see interacts with the plasma of stellar atmosphere, we need a tool to describe this process and to interpret the observed data we get.

We will call model of stellar atmosphere a self-consistent mathematical model of stellar atmosphere which contains the information about the distribution of main physical quantities (T,P,...) with geometrical depth counting from some zero-level.

Self-consistency means that having a set of free parameters the obtained solution is unique.

Importance of Model Atmospheres

1. Model atmospheres provide a link between theoretical and observational astrophysics
2. Model atmospheres are the upper boundary conditions for modeling of stellar structure and evolution
3. Model atmospheres are the intermediate region between stellar envelopes and interstellar medium
4. Model atmosphere is the only tool now for abundance analysis in wide range of physical conditions that is important for studying the chemical evolution of our Galaxy and Universe as a whole

Model Parameters

Energy Conservation

Since there are no energy sources in stellar atmosphere, the total amount of radiative energy falling at the bottom of the atmosphere from the inside should be the same at each point throughout the atmosphere:

$$div(F_{\mathrm{rad}}) = 0$$

A black-body Concept

A useful concept is that of black-body, as defined as a perfect absorber and emitter of light. It radiates energy at the same rate as it is being absorbed. The ratio of emitted and absorbed energy at each frequency depends only on temperature and is known as Plank function

Model Parameters

$$\frac{emission}{absorption} = B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

The total energy radiated by a black-body goes as a fourth power of temperature known as Stefan-Boltzmann Law:

$$\int_0^\infty B_\nu(T)d\nu = const \times T^4$$

If the radiation at the bottom of the atmosphere is black-body, then we can introduce a quantity called effective temperature T_eff so that at each atmospheric layer we have:

$$\int_0^\infty F_\nu d\nu = const \times {T_\mathrm{eff}}^4$$

T_eff is the temperature of black-body which radiates (from the unit surface area) the same amount of energy as the star of interest and thus is an energetic characteristic of the star

Model Parameters

Most of the stars do not change their radii on time scales compared with their life-times. Thus, we assume that the atmosphere of the star is in hydrostatic equilibrium, i.e. the gravitational force is balance by kinetic properties of plasma.

$$r_{\mathrm{atm}} \ll R_{\mathrm{star}}, \qquad g = \frac{GM_{\mathrm{star}}}{R_{\mathrm{star}}}$$

Stars with extensive atmospheres

Giants, supergiants, hot stars with strong stellar wind, etc.: R, M

Stars with extended atmospheres

Pulsating stars, supernovas, etc.: should be considered in hydrodynamic (t-dependance)

Model Parameters

Properties of radiation field and plasma in stellar atmosphere depend upon chemical composition (abundances) used $\{\varepsilon_i\}$

1. abundance of each element is given relative to the total number of atoms of all chemical elements (N_i/N_{total})
2. abundance of each element is given relative to the number of atoms of all chemical elements in a volume contains 10^{12} atoms of hydrogen
3. abundance of each element is given relative to the total mass of stellar plasma in unit volume

Self-consistency in spectroscopy

Model atmosphere → abundances → model atmosphere...

"Classical" Assumptions

- Plane-parallel geometry ($r_{\rm atm} \ll R_{\rm star}$). All the physical quantities depend only on geometrical depth h
- Homogeneous abundances
- Hydrostatic equilibrium (no large-scale motions)
- The atmosphere is time-independent (statistical equilibrium)
- There are no sources or sinks of energy
- Energy transport takes place only by radiation and convection (no heat conduction, acoustic waves, MHD waves, etc.)
- The free electrons as well as the free heavier particles obey the Maxwell distribution with local kinetic temperature $T_{\rm e}$.
- Local Thermodynamical Equilibrium (LTE) is assumed

Hydrostatic Equation

$$\frac{dP_{\rm total}}{dM} = g, \qquad dM = -\rho dx$$

$P_{\rm total}$ contains:

- Gas pressure $P_{\rm gas} = nkT$
- Radiation pressure $P_{\rm rad} = \int g_{\rm rad}(M) dM, g_{\rm rad} = \frac{4\pi}{c} \int \kappa_\nu H_\nu d\nu$
- Micro-turbulent pressure $P_{\rm turb} = \frac{1}{2}\rho\xi_{\rm turb}$
- Convective pressure $P_{\rm conv}$
- Pressure due to rotation $P_{\rm rot} = \int \frac{v^2}{R} dM$
- Macro-turbulent pressure $P_{\rm macro}$
- Magnetic pressure $P_{\rm mag}$

State Equation

Boltzmann formula for the ratio of occupation numbers:

$$\frac{n_i}{n_j} = \frac{g_i}{g_j} \exp(-\frac{E_j - E_i}{kT})$$

Saha equation for the ratio of ion numbers:

$$\frac{n_I}{n_J} = \frac{2}{n_\mathrm{e}} \frac{g_I}{g_J} (\frac{2\pi mkT}{h^2})^{3/2} \exp(-\frac{E_J - E_I}{kT})$$

Note

$n_\mathrm{e} = f(T, P, \epsilon_i)$ and is unknown. However, we know N_total and thus should solve the system of non-linear equations to find n_e. Iterative solution is needed.

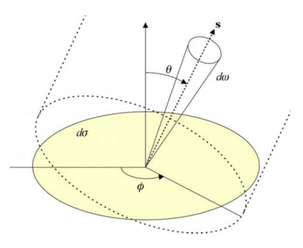
Equation of Radiative Transfer

Going through gas of the atmosphere, the radiation is absorbed, re-emitted and scattered many time. These define the way how the radiation at each frequency is transferred through the atmospheric region.

$$I_\nu = \frac{dE_\nu}{\cos\theta d\sigma d\nu d\omega dt}$$

$$[I_\nu] = erg\, cm^{-2}\, s^{-1}\, Hz^{-1}\, sterad^{-1}$$

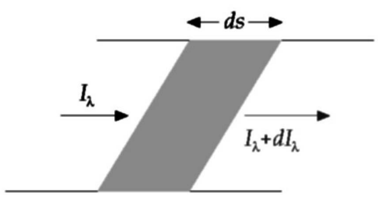
Number of photons times $h\nu$ per $d\nu d\omega dt$ and per $\cos\theta d\sigma$



Equation of Radiative Transfer

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$

Source function: $S_\nu = \frac{\eta_\nu}{\kappa_\nu}$
Optical depth: $d\tau_\nu = -\kappa_\nu \rho ds$
$\mu = \cos\theta$



Formal solution of RTE is:

$$I_\nu(\mu) = -e^{\tau_\nu/\mu} \int_c^{\tau_\nu} S_\nu e^{-t/\mu} \frac{dt}{\mu}$$

Moments of RTE

In model atmospheres we are meanly interested in integral characteristics of radiation field

$$J_\nu = \frac{1}{4\pi}\int I_\nu d\omega, \qquad H_\nu = \frac{1}{4\pi}\int \mu I_\nu d\omega, \qquad K_\nu = \frac{1}{4\pi}\int \mu^2 I_\nu d\omega$$

J_ν – mean intensity is an energy emitted by unit surface area in any direction

H_ν – radiative flux is an energy emitted by unit surface area in all directions

K_ν – is called K-integral and does not have physical meaning

However: $P_\nu^{(rad)} = \dfrac{4\pi}{c} K_\nu$ – radiative pressure acting on unit surface area

Moments of RTE

Source Function

It is seen that to solve RTE one needs to know source function S_ν which also depends upon the solution of RTE due to additional scattering term $\sigma_\nu J_\nu$

$$\mu\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu, \qquad \frac{dH_\nu}{d\tau_\nu} = J_\nu - S_\nu, \qquad \frac{dK_\nu}{d\tau_\nu} = H_\nu$$

$$S_\nu = \frac{\kappa_\nu B_\nu + \sigma_\nu J_\nu}{\kappa_\nu + \sigma_\nu}$$

Diffusive approximation ($\tau_\nu \gg 1$): $S_\nu(t_\nu) = \sum_{n=0}^{\infty} \frac{1}{n!}\frac{d^n B_\nu}{d\tau_\nu^n}(t_\nu - \tau_\nu)^n$

$$J_\nu(\tau_\nu) \approx B_\nu(\tau_\nu), \qquad H_\nu(\tau_\nu) \approx \frac{1}{3}\frac{dB_\nu(\tau_\nu)}{d\tau_\nu}, \qquad K_\nu(\tau_\nu) \approx \frac{1}{3}B_\nu(\tau_\nu)$$

Radiative Equilibrium

$$\frac{dH}{dM} = \int (\kappa_\nu + \sigma_\nu) J_\nu d\nu - \int (\kappa_\nu + \sigma_\nu) S_\nu d\nu = Q(M)$$

$Q(M)$–function describing the losses of radiative energy via its transformation to other types

Our experience shows that in most cases $Q(M) \approx 0$ so that the amount of absorbed energy is equal to that of emitted:

$$\int (\kappa_\nu + \sigma_\nu) J_\nu d\nu = \int (\kappa_\nu + \sigma_\nu) S_\nu d\nu$$

$$\frac{dH}{dM} = 0$$

Convective Energy Transport

Schwarzschild instability criteria

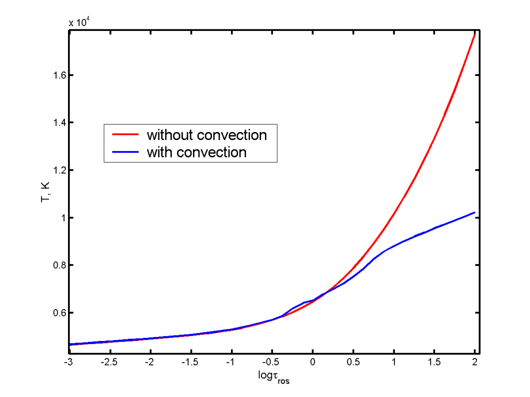
$$\frac{\gamma - 1}{\gamma}\left(-\frac{d\ln P}{dr}\right)_{\mathrm{R}} < \left(-\frac{d\ln T}{dr}\right)_{\mathrm{R}}, \qquad \gamma = \frac{C_{\mathrm{p}}}{C_{\mathrm{v}}}$$

Convection take place in the regions with high opacity due to fast ionization of main plasma components (H, He)

- Early type of stars: H is completely ionized so we can expect only weak convective zones due to He, He^{+} ionizations
- A-type: hydrogen convective zone close to photosphere
- F-type: more strong and deep hydrogen convective zone
- G-type: convection transfers most of the energy in deep layers and penetrate surface layers
- M-type: convection rules the whole structure of the atmosphere

Convective Energy Transport

Introducing convection in model atmosphere calculations we are mainly interested in which per sent of the total energy is carried out by convection



Convection Treatment

Mixing-Length Theory (MLT)

Based on local pressure scale height:

$$H_p = \frac{P_{total}}{\rho g} = \frac{1}{\alpha}$$

α has to be tuned for different stars

Application: cool stars atmosphere with strong convection regime

CM convection

Canuto & Mazzitelli (1991, 1992) – further improvement of convection model: no need for such free parameter as α any more

Application: atmospheres of A–F stars with weak convection regime

Temperature Correction

Problem

$T(M)$ which satisfies the conditions of RE is initially unknown

$$T(M) = T_0(M) + \Delta T(M)$$

$$\int (\kappa_\nu + \sigma_\nu) S_\nu(T_0 + \Delta T) d\nu = \int (\kappa_\nu + \sigma_\nu) J_\nu d\nu$$

$$S_\nu(T_0 + \Delta T) \approx S_\nu(T_0) + \frac{\partial S_\nu}{\partial T} \Delta T$$

$$\Delta T \approx \frac{\int (\kappa_\nu + \sigma_\nu)(J_\nu + S_\nu) d\nu}{\int (\kappa_\nu + \sigma_\nu) \frac{\partial S_\nu}{\partial T} d\nu}$$

Convergence Criteria

- Optically thick layers
  $(H_{rad} + H_{conv}) - \sigma T_{eff}^4 = 0$
- Optically thin layers
  $$\int \kappa_\nu J_\nu d\nu - \int \kappa_\nu S_\nu d\nu = 0$$

Convergence criteria:

1. ε(flux)$\leqslant$ 1%
2. ε(rad. equilibrium)$\leqslant$ 1%
3. $\Delta T_i \leqslant 1\,\mathrm{K}$ at each layer

Definition of the Problem

1. All equations are bound
   $S_\nu, J_\nu, H_\nu, K_\nu \rightarrow f(\alpha_\nu, \eta_\nu)$
   $\alpha_\nu, \eta_\nu \rightarrow f(T, P)$
   $T, P \rightarrow f(S_\nu, J_\nu, H_\nu, K_\nu)$
2. The system of equations is non-linear
   $J_\nu \rightarrow S_\nu \rightarrow J_\nu \ldots$
3. The system of equations is globally dependent throughout the atmosphere. Scattering processes force the properties of radiation field and plasma at one given point to be dependent from all other points.

Simple Model Atmosphere Calculation Flow Block

T_{eff}, $\log g$ (or R, M), $\{\epsilon_i\}$, ξ_{turb}

First approximation: $T(\tau_{\mathrm{std}})$

Hydrostatic: $P(\tau_{\mathrm{std}})$

State Equation: n_e, n_i, n_0

Opacity: α_ν^{cont}, α_ν^{line}

Radiation Field: J_ν, H_ν, K_ν, S_ν

ν–integration

Convection: H_{conv}

T-correction: ΔT

$T_{\mathrm{new}} = T_{\mathrm{old}} + \Delta T$

Model Iteration

Interaction Between Matter and Radiation

Absorption

- True Absorption
  Atom gets photon, goes to exited state and hits another atom transmitting its energy to the later one.
  Plasma heating
- Negative Absorption (or Stimulated Emission)
  Atom is exited by particle collision and emits the photon of energy $h\nu$ in direction ω
  Plasma cooling

Interaction Between Matter and Radiation

Emission

Atom *spontaneously* emits a photon due to finite life-time. The atom should be in exited state (whatever was the reason).

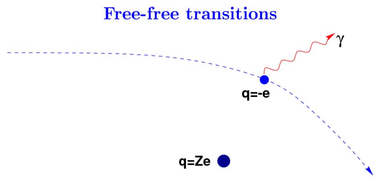
Scattering

- Direct Scattering
  Atom gets photon from the beam, goes to exited state and then re-emits its energy in arbitrary direction. Depending upon the probability of direction of the re-emitted photon we have *isotropic* and *anisotropic* scattering. If the energy of re-emitted photon is equal to that of absorbed one, we have *coherent* scattering. Otherwise *incoherent*.
- Reverse Scattering
  The photon is added to the beam due to direct scattering of another beam on a given atom

Interaction Between Matter and Radiation

Absorption and Emission processes can take place between the following energy levels:

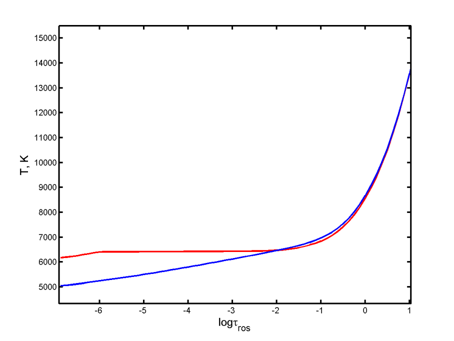
- discrete-discrete (bound-bound) $\Rightarrow$ spectral lines
- discrete-continuous (bound-free) $\Rightarrow$ ionization/recombination continua
- continuous-continuous (free-free) $\Rightarrow$ ion required



Line Blanketing and Model Atmospheres

Opacity due to atomic lines absorption is one of the most important effects in stellar atmospheres

Deep layers	–	backwarming
Surface layers	–	cooling



Line Opacity Treatment

Problem

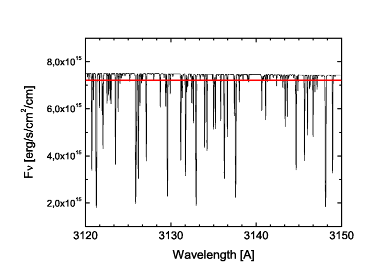
Hundred of thousands of lines which have to be taken into account

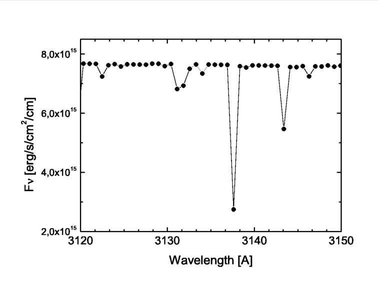
Solution

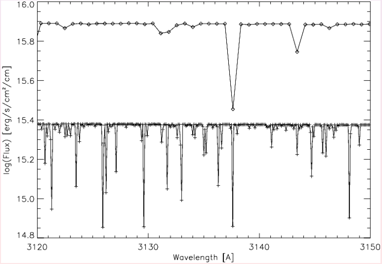
Several statistical methods have been developed

- Opacity Distribution Function (ODF)
- Opacity Sampling (OS)
- Direct Opacity Sampling

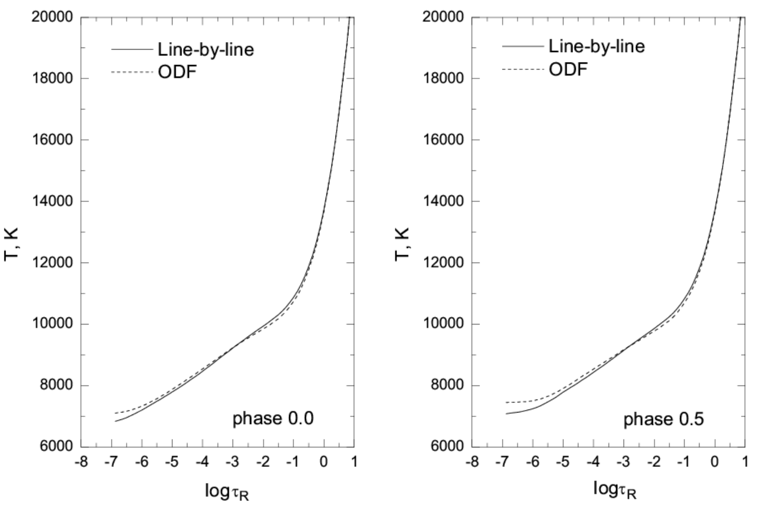
Line Opacity Treatment



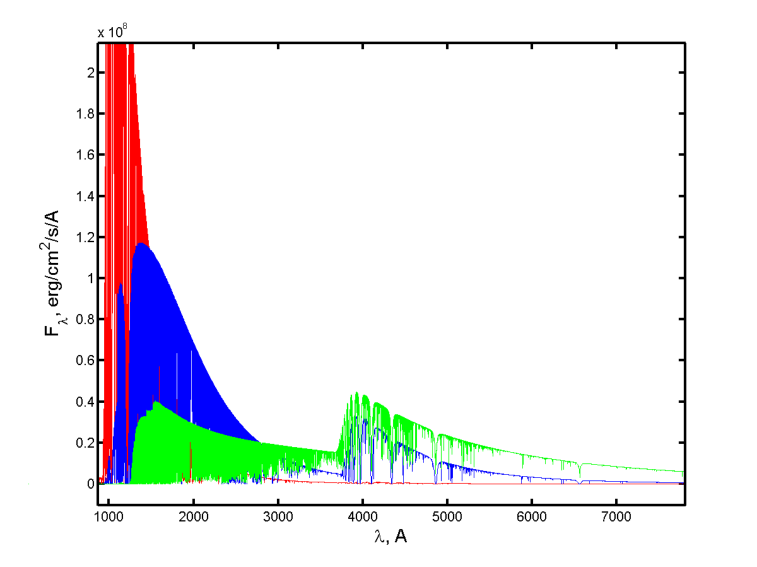




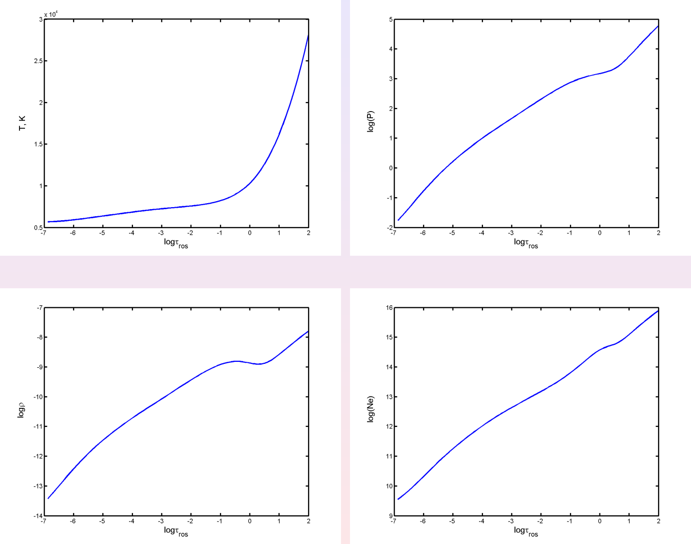
Line Opacity Treatment



Energy Redistribution



Model Structure



Model Atmospheres in Spectroscopy

Fundamental parameters $\longrightarrow$ model atmosphere $\longrightarrow$ synthetic spectra $\longrightarrow$ physical output

For most spectroscopists model atmosphere is a black box.

- Learn how to use existing codes by yourself
- Use already calculated public model grids
- If you are lazy, ask other people to calculate models for you (not always work perfectly)

Self-consistency in Spectroscopy

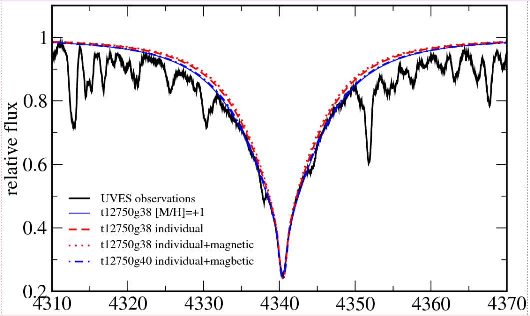
Model atmosphere is consistent with observations if the following observables are fitted simultaneously:

- Energy distribution
  Contains information about total energy balance
- Hydrogen line profiles
  Cool stars: most sensitive to temperature changes
  Hot stars: most sensitive to pressure changes
- Metallic line spectra
  Abundances (stratification, spots), micro-turbulence, rotation, magnetic fields, etc.

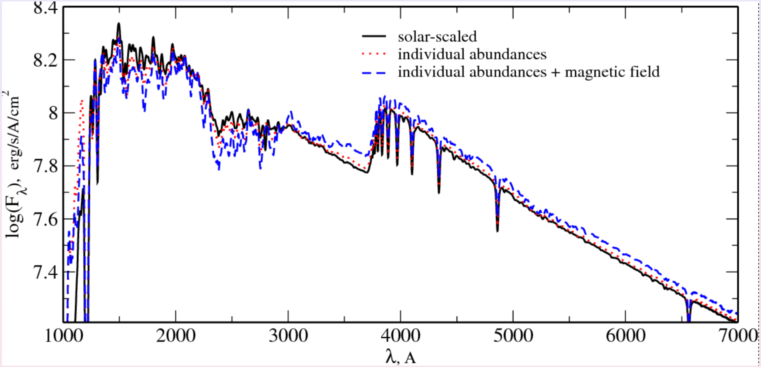
Self-consistency in Spectroscopy

Once abundances and other physical properties have been derived it is necessary to recalculate the model atmosphere to consistency consistency

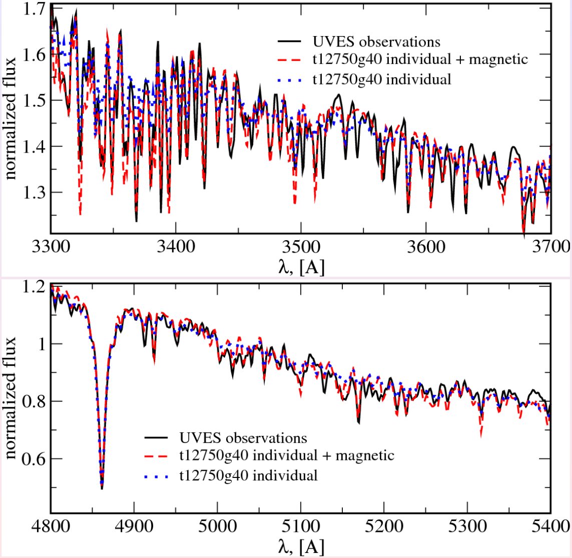
Iterative solution is needed



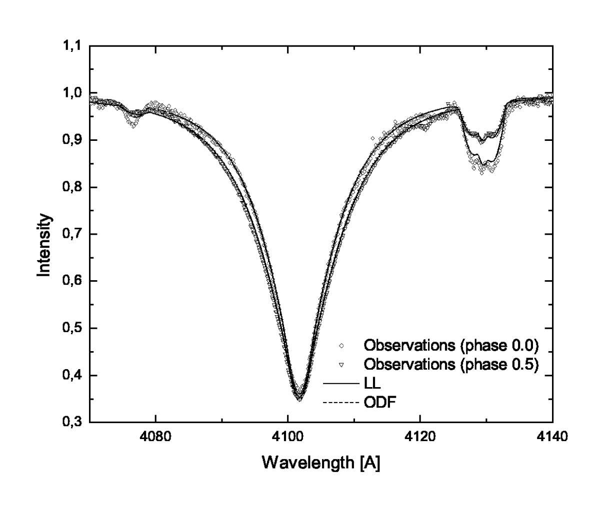
Self-consistency in Spectroscopy



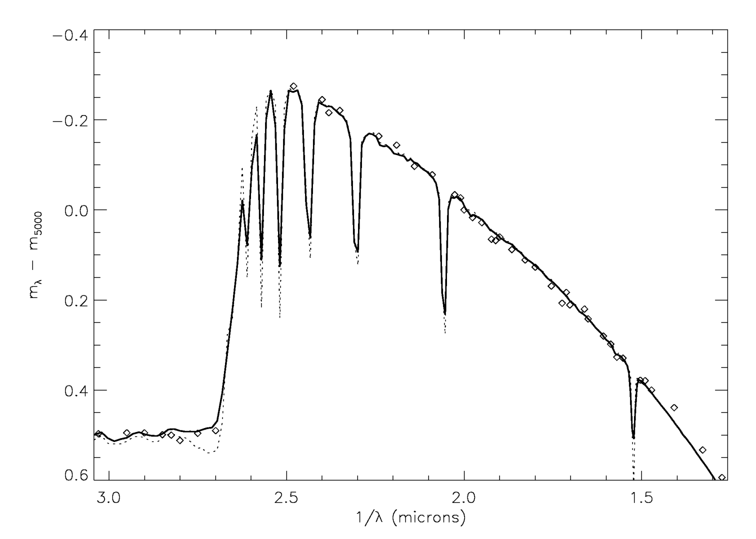
Self-consistency in Spectroscopy



Self-consistency in Spectroscopy



Self-consistency in Spectroscopy



non-LTE

In almost all plasmas (except very thin one with magnetic fields), the Maxwell distribution is valid. The validity of Saha and Boltzmann formulas on the other hand depend on the ratio of photon number density $\varepsilon_{\rm photons}$ to the particle number density $\varepsilon_{\rm particles}$

$$\varepsilon_{\rm photons} = \frac{4\pi}{c}\int S_\nu d\nu = \frac{4}{c}\sigma_{\rm B} T^4, \qquad \varepsilon_{\rm particles} = \frac{3}{2}nkT$$

$$f = \frac{\varepsilon_{\rm photons}}{\varepsilon_{\rm particles}} = 36.5\frac{T^3}{n}$$

- $f \ll 1$: LTE
- $f \sim 1$: LTE questionable
- $f \gg 1$: non-LTE

non-LTE

$$\frac{dn_i}{dt} = -n(i)\sum_j [R_{(i\to j)} + C_{(i\to j)}] + \sum_j n(j)[R_{(j\to i)} + C_{(j\to i)}] = 0$$

Upward rates, bound-bound and bound-free:

$$R_{up}(i \to j) = \int 4\pi\alpha_{nu}\frac{J_\nu}{h\nu}d\nu$$

Downward rates, bound-bound, free-bound

$$R_{down}(i \to j) = \frac{n_j}{n_i}\int \alpha_\nu e^{-h\nu/kT}\left(\frac{4\pi}{h\nu}J_\nu + \frac{8\pi\nu^2}{c^2}\right)$$

$$b_i^{Zwaan} \equiv n_i/n_i^{LTE}, \qquad b_i^{Menzel} \equiv \frac{n_i/n_i^{LTE}}{n_c/n_c^{LTE}}$$

n_c–concentration of next ionization stage

Validity of LTE

LTE	LTE questionable	Non-LTE
• Stellar atmospheres ($n = 10^{13}...10^{16}$ cm^{-3}) for $T_{\text{eff}} \sim 25000$ K • Stellar cores	• Atmospheres of super-giants ($n = 10^{10}...10^{13}$ cm^{-3})	• Outer atmospheres (chromosphere,coronae) • Stellar winds • Interstellar medium • Intergalactic medium

In stellar model atmospheres we are mainly interested in non-trace elements like H, He, Fe, etc. (most abundant species having many strong lines in the spectra).

Additional Physics

Additional physics have to be included to improve models and our understanding about outer regions of the stars and to model their spectra

- Spherical symmetry: stars with extended atmospheres
- Stellar wind: hot stars (multi-component stellar wind)
- Diffusion of chemical elements: CP stars with quite atmospheres (horizontal and vertical distribution)
- Magnetic fields: CP2 stars with strong magnetic fields
- Rotation: fast rotating hot stars, meridional circulations
- Binarity effects: dust disks, x-ray emission, tidal effects, etc.
- ...

Model Grids

- ATLAS9, ATLAS12 (R.L. Kurucz)
  http://kurucz.harvard.edu
- TLUSTY (Lanz T., Hubeny I.)
  http://nova.astro.umd.edu/index.html
- MARCS (Gustafsson B. et al.)
  http://marcs.astro.uu.se
- NEMO grid based on ATLAS9 with different convection treatment
  http://ams.astro.univie.ac.at/cgi-bin/dive.cgi

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