Definitions
&
terms

Content

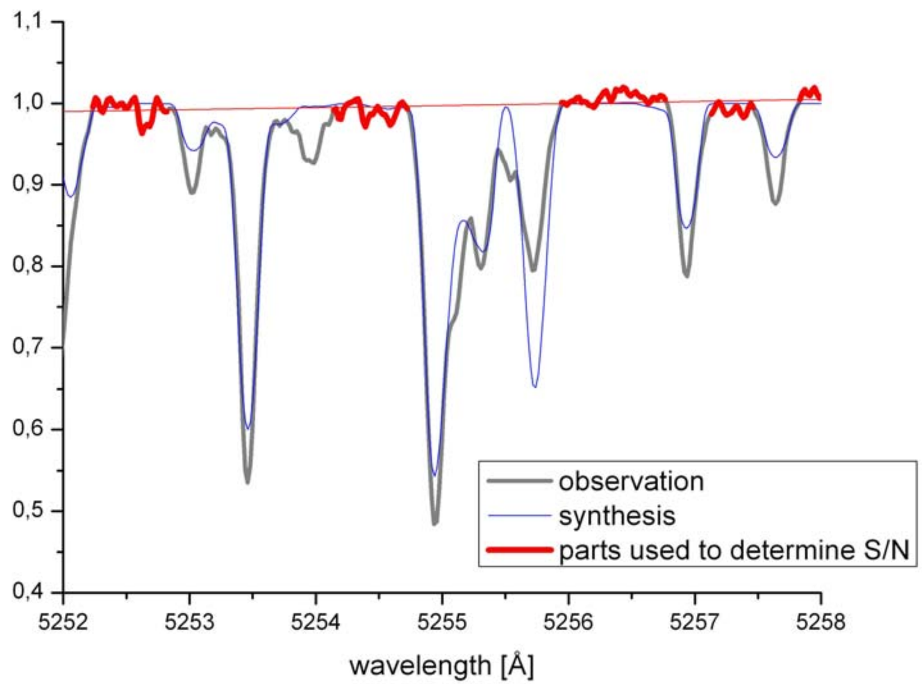
- Signal to noise ratio (SNR)
- Resolution (R)
- Continuum normalization problems
- Telluric lines
- Line shape
- Equivalent width (EQW)
- Bisectors
- Radial velocity (v_{rad})
- Rotational velocity (vsin*i*)
- Temperatures (T)
- Gravity (log*g*)
- Macroturbulence (ζ_{RT})
- Microturbulence (ξ)
- Optical depth (τ)

S/N

Signal to noise ratio should be calculated in the continuum.

$I_{mean} = 0.99808$
$\sigma = 0.01026$
$\Rightarrow SNR = I_{mean} / \sigma = 97.3$

- Use parts where „no“ lines are present
- Linear fit or, if normalization is good enough, $I_{mean} = 1$
- Calculate standard deviation σ
- $SNR = I_{mean} / \sigma$

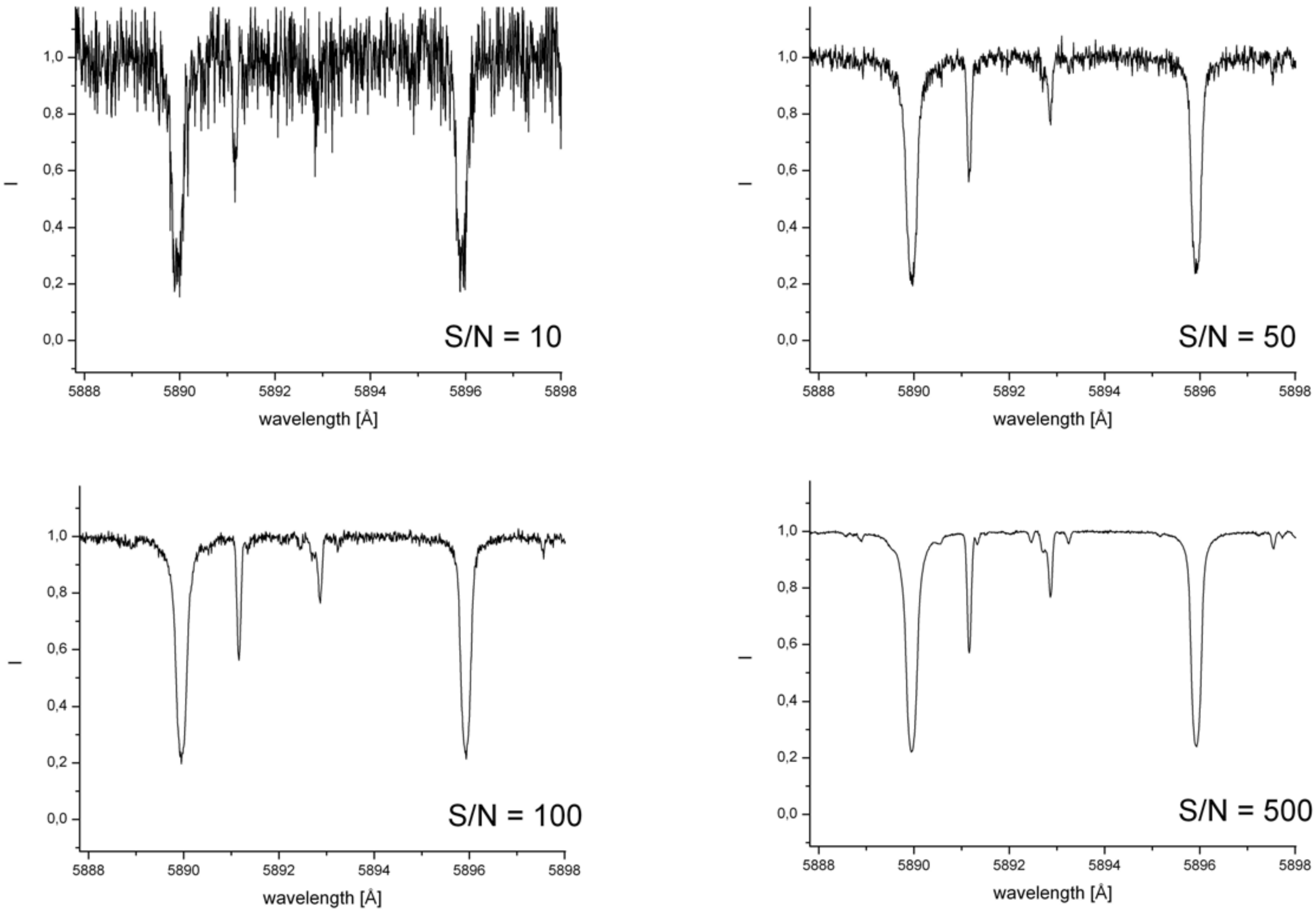


Ways to increase SNR:

- longer exposure time
- sum up spectra
- combine similar lines (LSD technique)

http://www.ast.obs-mip.fr/users/donati/multi.html
Least Squares Deconvolution (LSD) technique (Donati et al. 1997)

S/N


S/N = 10
S/N = 50
S/N = 100
S/N = 500
1,0
0,8
0,6
0,4
0,2
0,0
5888
5890
5892
5894
5896
5898
wavelength [Å]

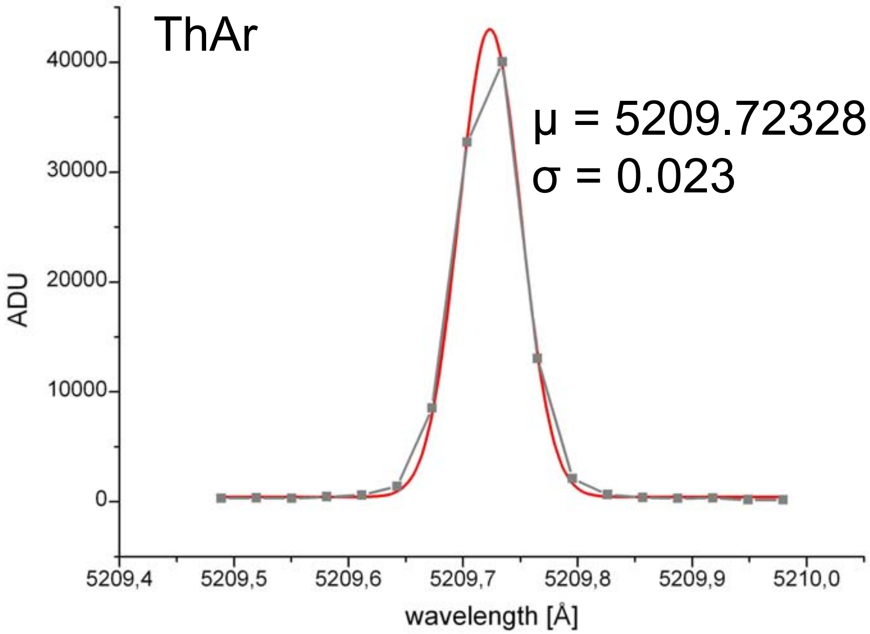
R

- Resolution:

$$R = \frac{\lambda}{\Delta\lambda}$$

$\Delta\lambda$ ... width of resolved spectral element

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$



μ = 5209.72328
σ = 0.023

$$FWHM = 2\sqrt{2\ln(2)}\,\sigma$$

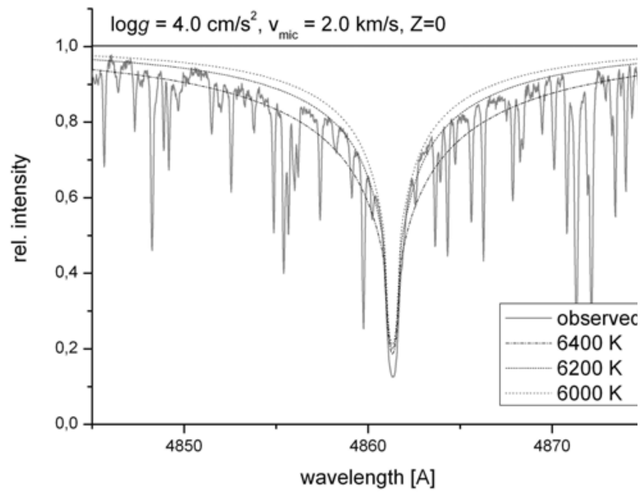
≈ 2.355 σ = 0.05414 Å
=> R = 96227

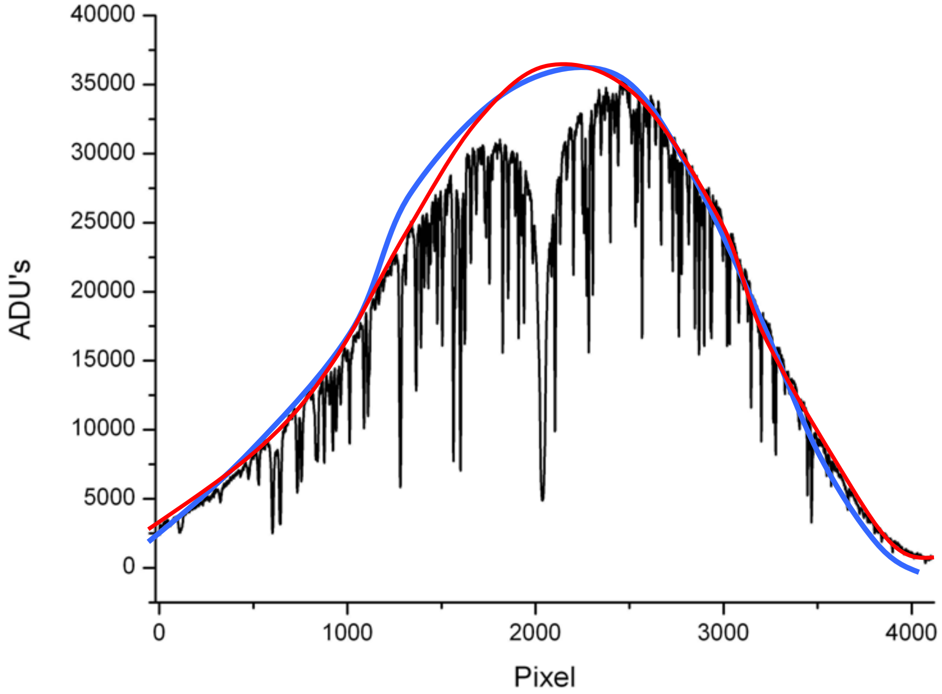
Continuum normalization

Hydrogen lines:

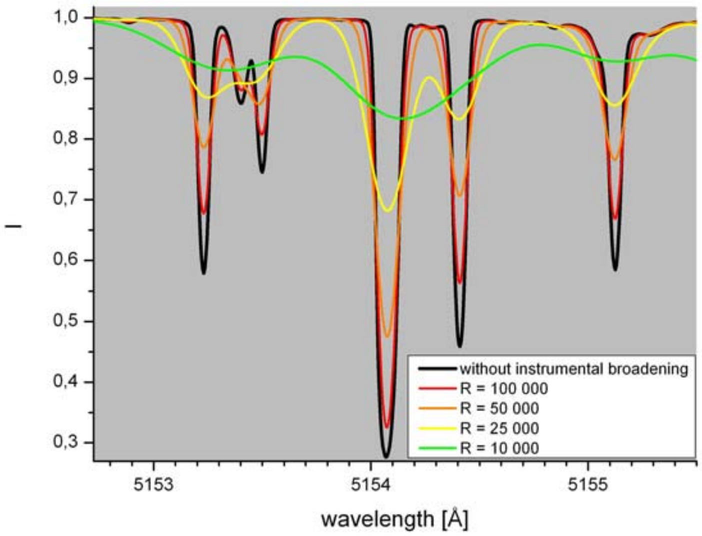
- Continuum normalization tricky
  (→ interpolate continuum between uncontaminated echelle orders;
  check quality of normalization by checking line symmetry)

- Errors in normalization procedure can
  cause errors in temperature determination
  (→ use other methods additionally)

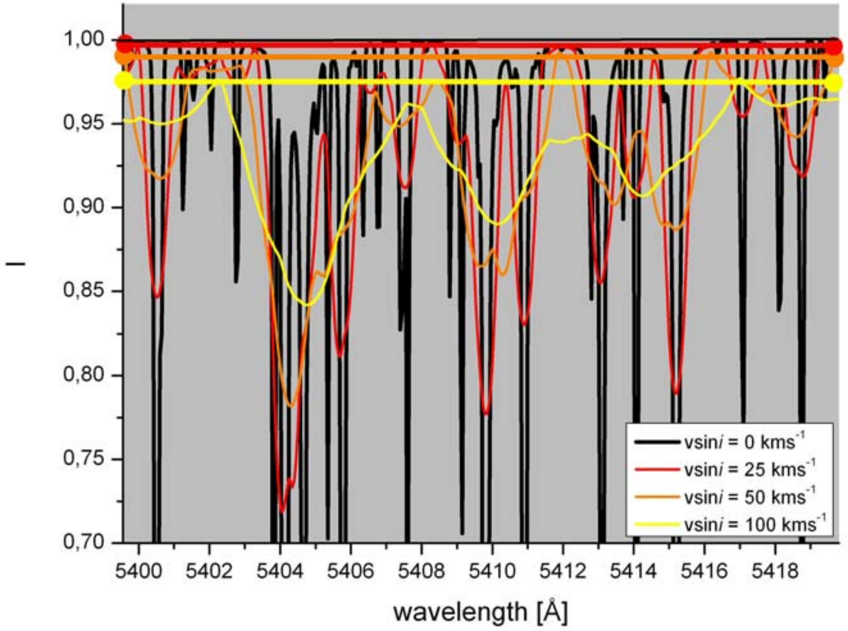




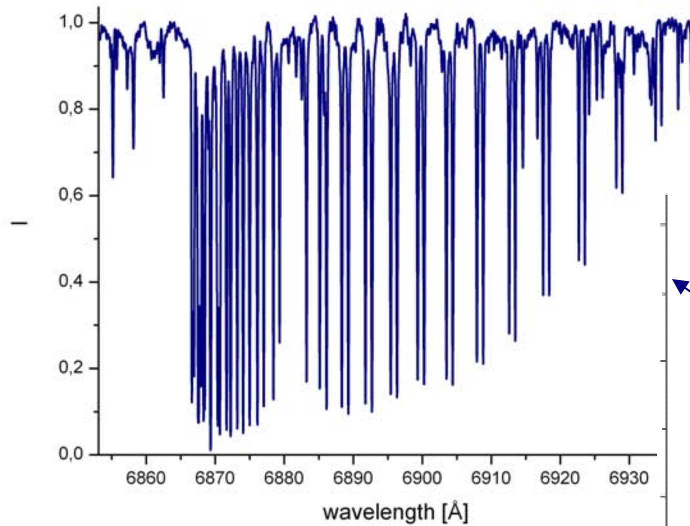
Continuum normalization



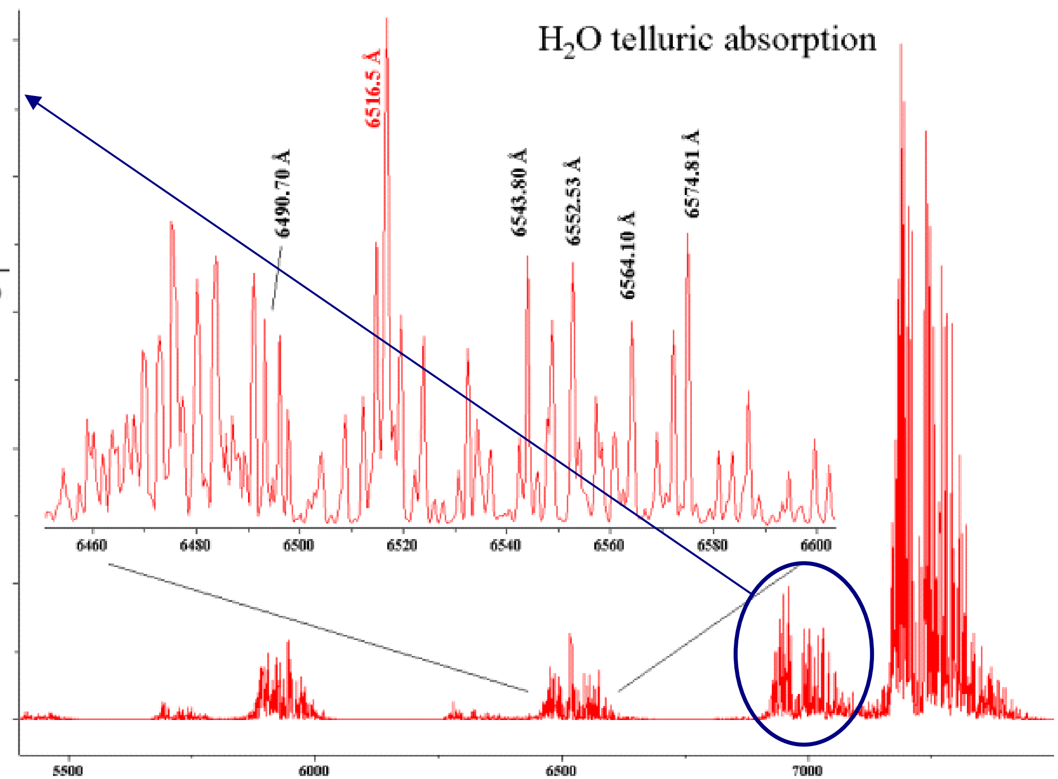
Line broadening leads to underestimation of continuum!



Telluric lines

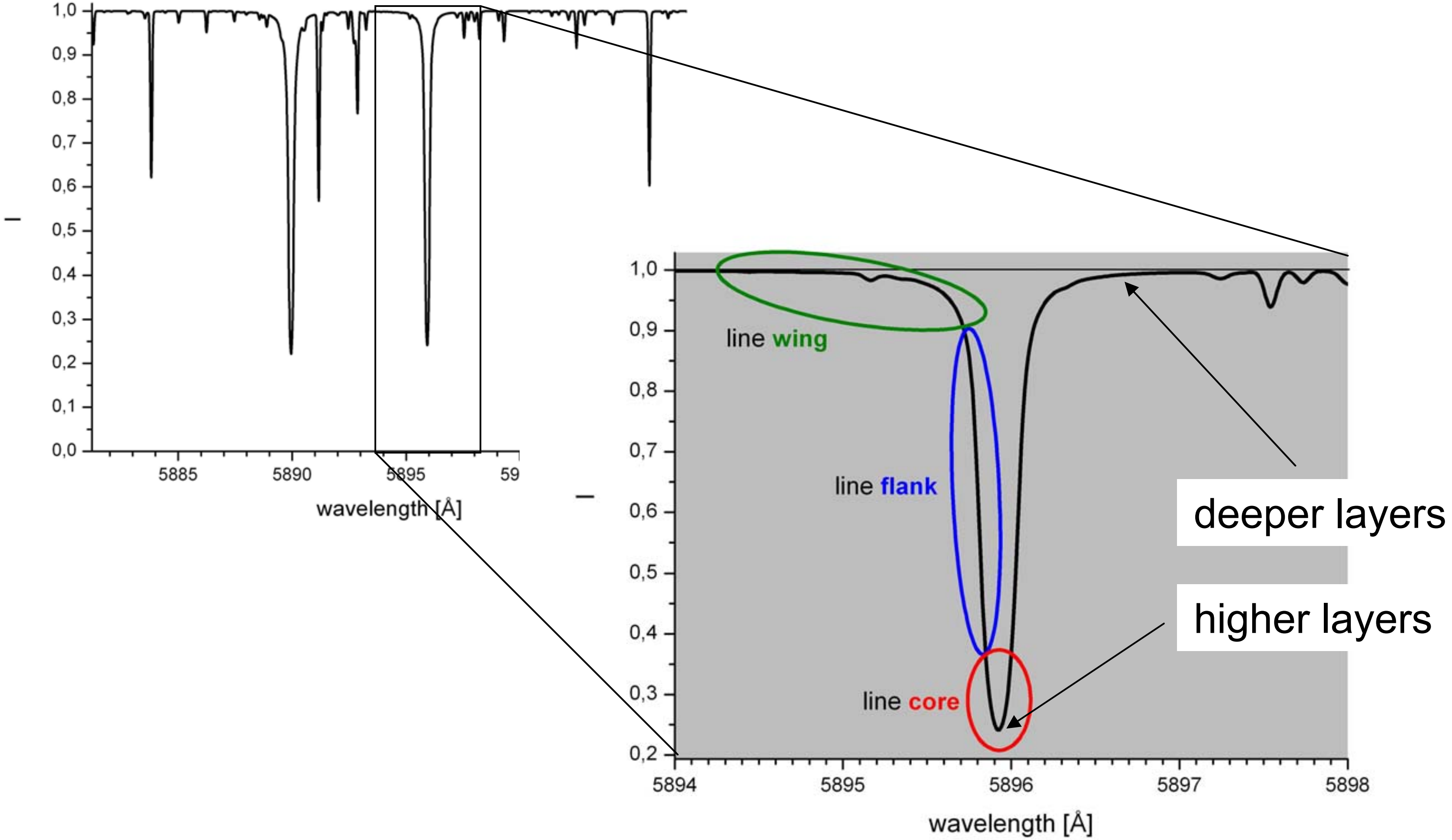


Telluric lines can contaminate spectrum!



H_20 telluric absorption: http://www.astrogeo.va.it/astronom/spettri/atmosferen.htm

Line shape



1,0
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0,6
0,5
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0,3
0,2
0,1
0,0
I
5885
5890
5895
59
wavelength [Å]
line wing
line flank
line core
deeper layers
higher layers
5894
5895
5896
5897
5898
wavelength [Å]

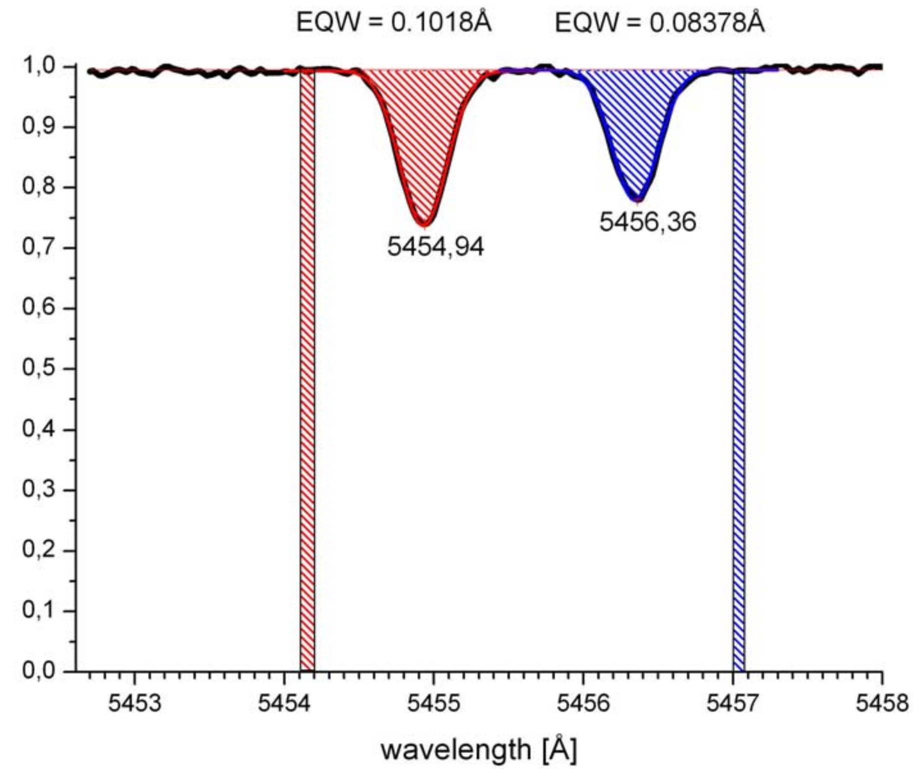
EQW

- The equivalent width is the width of a rectangle with height 1 centered on a spectral line that, on a plot of intensity against wavelength, has the same area as the line.

$$W_t = \int_{-\infty}^{+\infty} \frac{F_c - F_\nu}{F_c}\, d\nu$$

... true equivalent width

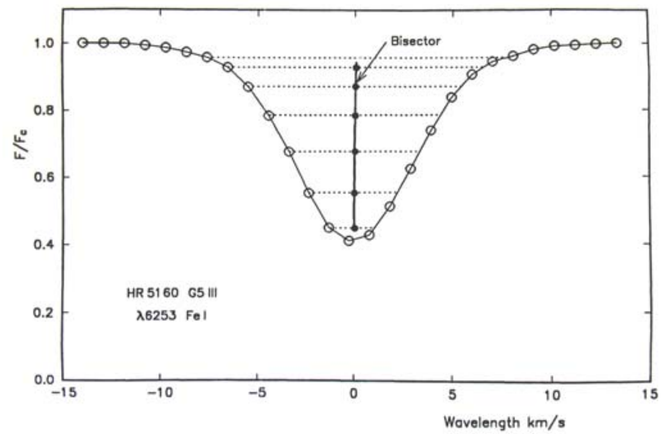
$$W_m = \int_{-\Delta}^{+\Delta} \frac{I(\lambda) * (F_c - F_\nu)}{D_c}\, d\nu$$

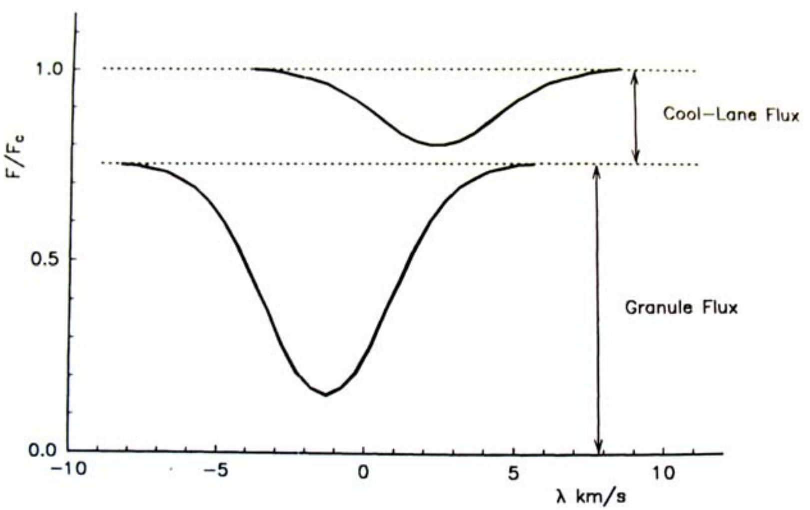


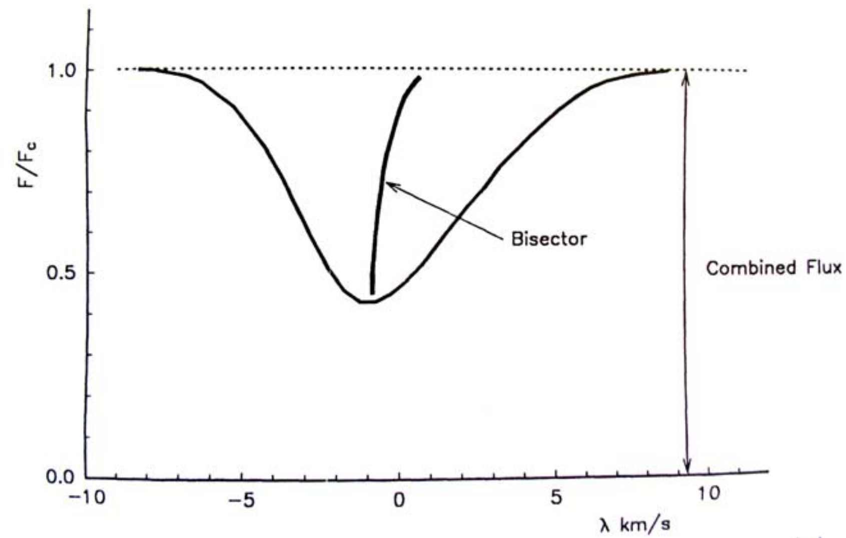
... measured equivalent width. $I(\lambda)$... instrumental profile

Bisectors

- Bisector connects midpoints of horizontal cuts through a line profile.
- Information about photospheric velocity fields.
- Rising (blue shifted) and falling (red shifted) material form asymmetric line profile.
- Cool stars: (
- Hot stars:)







v_{rad}

- Radial velocity

$$v_{rad} = \frac{v}{c} = \frac{\Delta\lambda}{\lambda}$$

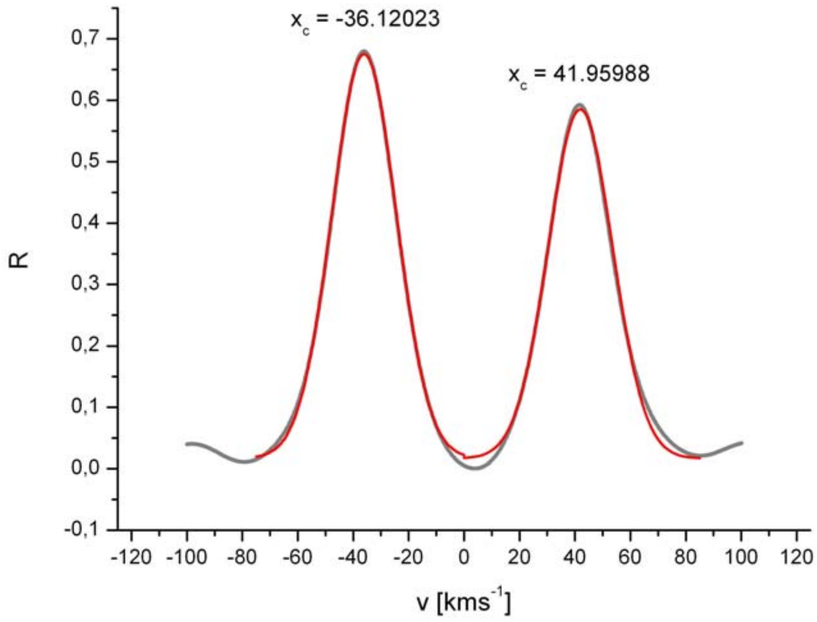
1a) Measure center of unblended lines over a large wavelength range.

1b) Synthisize spectrum and cross correlate it with your observation.

1c) Use LSD technique on unblended lines.

2) Remove earths‘ velocity contribution in the line of sight (e.g. with IRAF task „dopcor“)

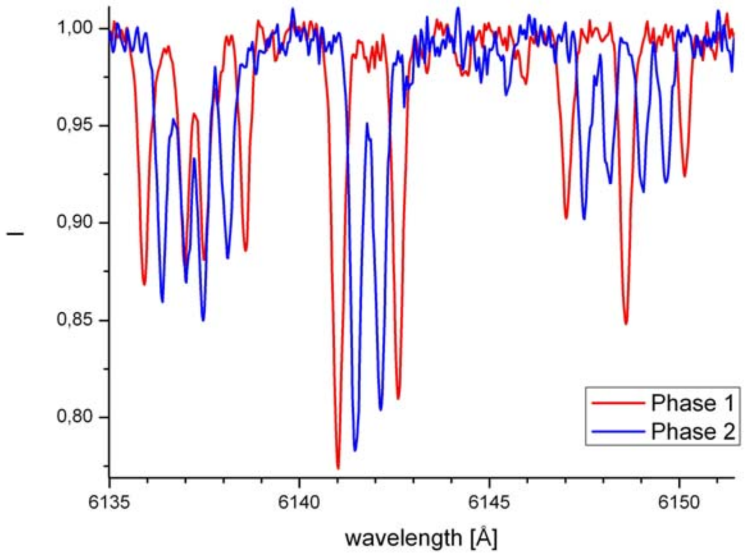
3) Correct your observation to laboratory wavelengths.



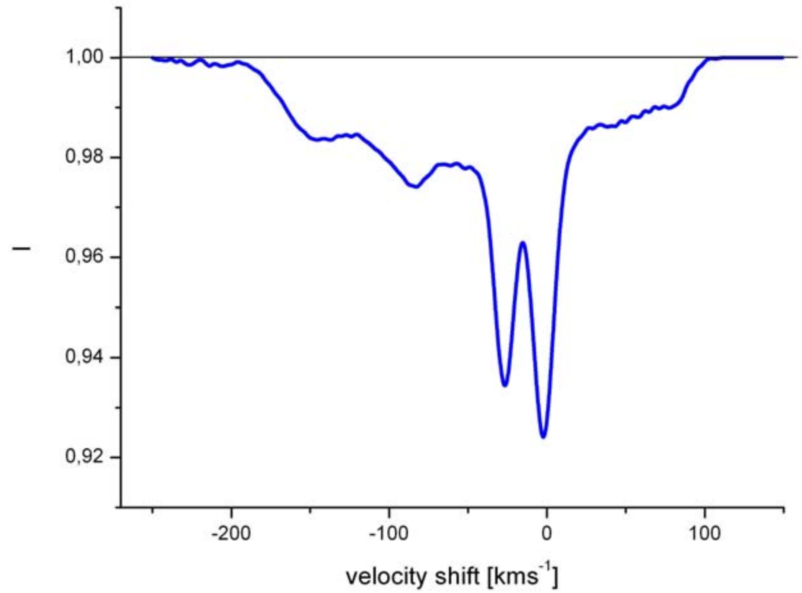
Binaries

- Radial velocity varies with time / phase.
- SB1 ...only one component is visible in spectrum
- SB2 ...both components are visible

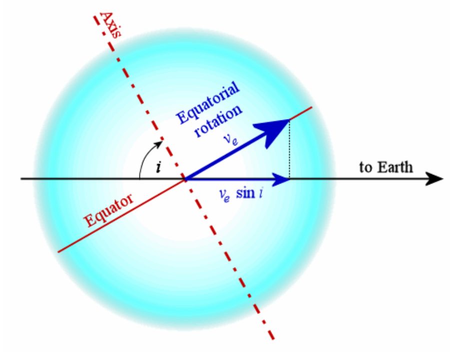
HD10167

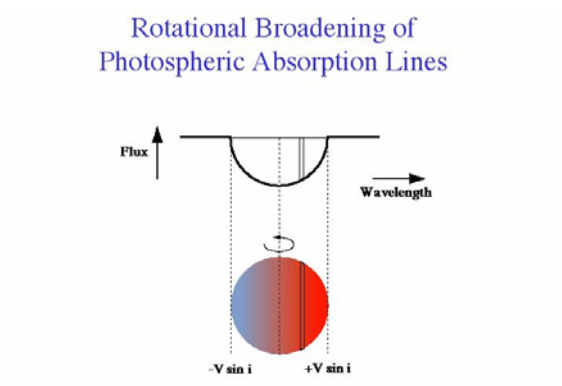


HD166114



vsin*i*





http://www.bartol.udel.edu/~owocki/RDOME/Swarthmore/

$$\mathbf{s}(\lambda) = \mathbf{h}(\lambda) * \mathbf{b}(\lambda) * \mathbf{p}(\lambda)$$

s... observed spectrum
h... „true“ spectrum
b... broadening function
p... instrumental profile

$$\mathbf{b}(\lambda') = c_1 \sqrt{1 - \lambda'^2} + c_2 (1 - \lambda'^2)$$

$$\lambda' = \frac{(\lambda - \lambda_0)}{\Delta\lambda}$$

$$\Delta\lambda = \frac{\lambda_0 \, \mathrm{vsini}}{c}$$

$$c_1 = \frac{2(1-\epsilon)}{\pi\Delta\lambda\left(1 - \frac{\epsilon}{3}\right)}$$

$$c_2 = \frac{\epsilon}{2\Delta\lambda\left(1 - \frac{\epsilon}{3}\right)}$$

λ-λ_0... velocity shift
ε... limb darkening coefficient

$$I(\Theta) = I_0[1 - \epsilon(1 - \cos\Theta)]$$

vsini_{sun}: 1.8 km/s

Temperature

- T_{eff}, T_{rad}

$$F = \frac{L}{4\pi R^2} = \sigma T_{eff}^4$$

- $T_{kinetic}$

$$n(v)\,dv = \left(\frac{m}{2\pi k T_{kin}}\right)^{\frac{3}{2}} e^{-\frac{mv^2}{2kT_{kin}}} 4\pi v^2\,dv$$

$$T_{kin} = \frac{2}{3}\frac{1}{k} < \frac{mv^2}{2} >$$

- $T_{excitation}$

$$\frac{n\,(\text{upper level})}{n\,(\text{lower level})} = \frac{g_u}{g_l} e^{\frac{\Delta E}{kT_{ex}}}$$

- $T_{ionization}$

$$\frac{N_{i+1} P_e}{N_i} = \frac{2kT_{ion} u_{i+1}}{u_i} \left(\frac{2\pi m_e k T_{ion}}{h^2}\right)^{\frac{3}{2}} e^{\frac{-\chi_i}{kT_{ion}}}$$

collision dominated, ideal gas

LTE: $T_{eff} = T_{kin} = T_{ex} = T_{ion}$

$T_{eff,sun}$: 5777 K

log*g*

- surface gravity → gas pressure ↔ electron pressure
- gravity ↔ gas pressure + radiation pressure + magnetic fields + velocity fields ≠ GM/R^2

a) Measuring lines of different species (neutral vs. single ionized)

b) Line wings (Van der Waals, Stark broadening) Hydrogen lines, Na I D lines (5890 Ǻ), Ca I

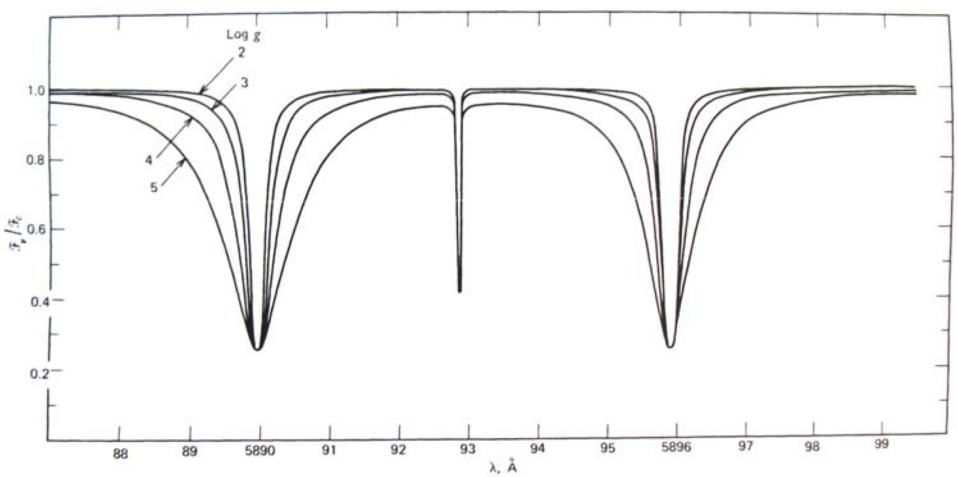
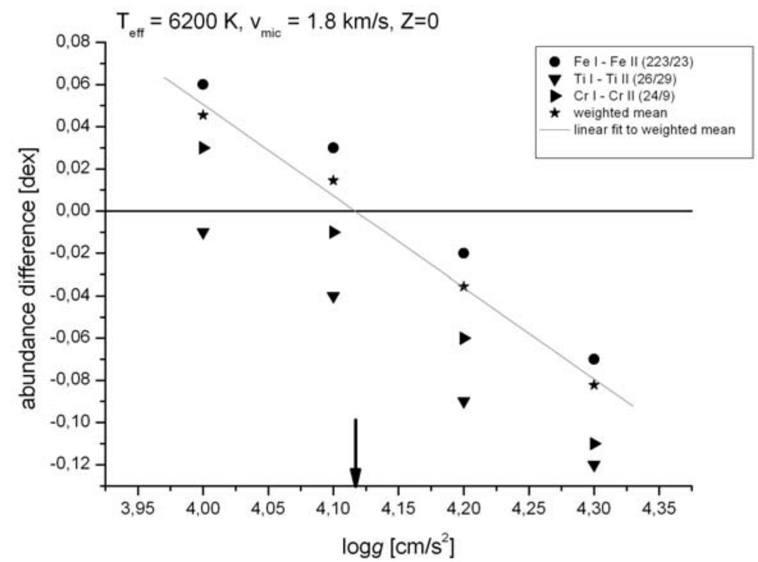


Fig. 16.3. The gravity dependence of the sodium D lines is shown for models having $S_0 = 1.0$ and gravity values as shown.



T_{eff} -effects can compensate logg-effects !

$\log g_{sun}$: 4.44 cm/s²

v_{mac} - ζ_{RT}

- Size of turbulent cell is large compared to mean free path of photon.
  (Photons remain in turbulence cells from creation to escape.)
- Disk integrated spectrum is sum of doppler shifted spectra created in turbulence cells.
- Equivalent width of line is conserved.
- Mactroturbulence is anisotropic -> radial and tangetial component (ζ_R, ζ_T)

$$F_\nu = \pi I_\nu \left[\frac{2 A_R \Delta\lambda}{\sqrt{\pi}\, \zeta_R^2} \int_0^{\frac{\zeta_R}{\Delta\lambda}} e^{-\frac{1}{u_1^2}} \, d u_1 + \frac{2 A_T \Delta\lambda}{\sqrt{\pi}\, \zeta_T^2} \int_0^{\frac{\zeta_T}{\Delta\lambda}} e^{-\frac{1}{u_2^2}} \, d u_2 \right]$$

$$u_1 = \frac{\zeta_R \cos\theta}{\Delta\lambda}$$

$$u_2 = \frac{\zeta_T \sin\theta}{\Delta\lambda}$$

$$A_R = A_T$$

$$\zeta_R = \zeta_T = \zeta_{RT}$$

$$F_\nu = \pi I_\nu \left[\frac{2 \Delta\lambda}{\sqrt{\pi}\, \zeta_{RT}^2} \int_0^{\frac{\zeta_{RT}}{\Delta\lambda}} e^{-\frac{1}{u^2}} \, d u \right]$$

$\zeta_{RT,sun}$: 2 km/s

v_{mic} - ξ

- Size of turbulent cell is small compared to mean free path of photon
- unsaturated line: microturbulence broadenes line but conserves equivalent width
- saturated line: ξ widens wavelength range covered by absorption => saturation reduced => increasing total absorption.

$$\alpha = \alpha' * N(\Delta\lambda)$$

α... line absorption coefficient broadened by microturbulence
α'... unbroadened line absorption coefficient

$$N(v)\,dv = \frac{1}{\sqrt{\pi}\,\xi} e^{-\frac{v^2}{\xi^2}}\,dv$$

...isotropic Gaussian

$$\Delta\lambda_D = \frac{\lambda}{c}\sqrt{\frac{2kT}{m} + \xi^2}$$

...thermal broadening

ξ_{sun}: 1.1 km/s

optical depth τ

- „Escape probability of a photon starting at dx under the surface“ OR „number of mean free paths traveled by a photon over a displacement dx“.
- „An optical depth of unity is that thickness of absorbing gas from which a fraction of 1/e photons can escape.“
- τ increases downward as z decreases. τ = 0 ...surface

$$dI_\nu = -\kappa_\nu \rho I_\nu \, dx$$

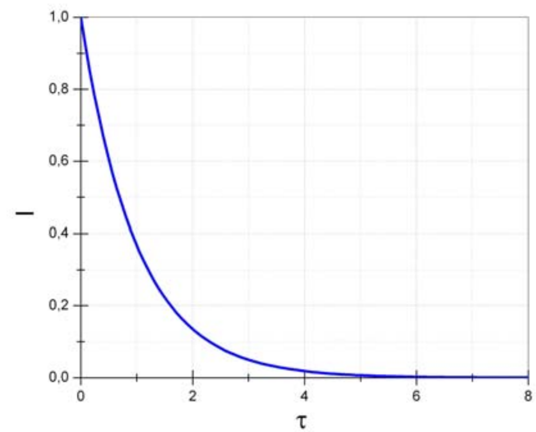
$$\tau_\nu = \int_0^L \kappa_\nu \rho \, dx$$

$$d\tau_\nu = -\kappa_\nu \rho \, dx$$

$$dI_\nu = -I_\nu \, d\tau_\nu$$

$$I_\nu = I_\nu^0 e^{-\tau_\nu}$$

τ	I / I0	dx (sun) [km]
-10,000	22026,466	-1515,2
-6,000	403,429	-909,1
-4,000	54,598	-606,1
-2,000	7,389	-303,0
-1,000	2,718	-151,5
0,000	1,000	0,0
0,100	0,905	15,2
0,250	0,779	37,9
0,500	0,607	75,8
0,667	0,513	101,0
1,000	0,368	151,5
2,000	0,135	303,0
4,000	0,018	606,1
6,000	0,002	909,1
10,000	0,000	1515,2



Gray atmosphere: $\kappa \neq f(\nu)$

$$T_x = \frac{1}{2} T_{eff}^4 \left(1 + \frac{3}{2}\tau\right)$$

$$T_x = T_{eff} \text{ at } \tau = \frac{2}{3}$$

Presentations

- Fourier analysis in spectroscopy
- Spectrophotometry
- Stellar rotation
- The Earth‘s atmosphere / telluric lines
- ...